

$$4a. y = a\sqrt{b(x-h)} + k$$

$$f(x) = \sqrt{x}$$

$$g(x) = f(x+5) - 2$$

$$\text{Vertex } (-5, -2) (h, k)$$

$$a = 2$$

$$\therefore y = 2\sqrt{x+5} - 2$$

$$b. y = a\sqrt{b(x-h)} + k$$

$$f(x) = \sqrt{x}$$

$$g(x) = f(x-8) - 6$$

$$\text{Vertex } (8, -6) (h, k)$$

$$a = 1$$

$$\therefore y = -1\sqrt{x-8} - 6$$

$$y = -1\sqrt{-(x-8)} - 6$$

$$5 \quad (0, 5) \quad (3, 1)$$

$$M = \frac{\Delta y}{\Delta x} = \frac{1-5}{3-0} = -\frac{4}{3}$$

$$-\frac{4}{3} = \frac{y-1}{x-3}$$

$$3y - 3 = -4x + 12$$

$$3y = -4x + 15$$

$$y = -\frac{4}{3}x + 5$$

$$\text{Domain } f(x): -\infty < x < \infty$$

$$\text{Interval } (-\infty, \infty)$$

$$\text{Range } f(x): -\infty < f(x) < \infty$$

$$\text{Interval } (-\infty, \infty)$$

$$\text{Domain } \sqrt{f(x)}: x \leq \frac{15}{4}$$

$$(-\infty, \frac{15}{4})$$

$$\text{Range } \sqrt{f(x)}: f(x) \geq 0$$

$$[0, \infty)$$

$$6. y = f(x)$$

$$y = (x-2)^2 - 7$$

$$\text{Domain } f(x): -\infty < x < \infty$$

$$\text{Interval notation } (-\infty, \infty)$$

$$\text{Range } f(x): f(x) \geq -7$$

$$\text{Interval notation } [-7, \infty)$$

$$y = \sqrt{f(x)}$$

$$y = \sqrt{(x-2)^2 - 7}$$

$$\text{Domain } \sqrt{f(x)}:$$

$$x \leq -\sqrt{7} + 2 \text{ or } x \geq \sqrt{7} + 2$$

$$\sqrt{(x-2)^2 - 7} = 0$$

$$(x-2)^2 - 7 = 0$$

$$(x-2)^2 = 7$$

$$x-2 = \pm\sqrt{7}$$

$$x = -\sqrt{7} + 2 \text{ or } \sqrt{7} + 2$$

$$\text{Domain } \sqrt{f(x)}$$

$$= x \leq -\sqrt{7} + 2 \text{ or } x \geq \sqrt{7} + 2$$

$$(-\infty, -\sqrt{7} + 2] \cup [\sqrt{7} + 2, \infty)$$

$$\text{Range } \sqrt{f(x)}:$$

$$f(x) \geq 0$$

$$[0, \infty)$$

$$9. y = \frac{3x-4}{x-2}$$

i) Vertical asymptote

$$x-2=0$$

$$x=2$$

$$ii) \frac{3x-4}{x-2}$$

$$\frac{3x-4(x-2)}{x-2} = \frac{3x-4x+8}{x-2}$$

$$\frac{-x+8}{x-2} \text{ or } \frac{8-x}{x-2}$$

$$x-2=0$$

$$x=2$$

$\therefore$ NPVs: $x=2$

iii) x-intercept, $y=0$

$$\frac{8-x}{x-2} = 0$$

$$x-2$$

$$8-x=0(x-2)$$

$$8-x=0$$

$$8=x$$

$$\text{or } x=8$$

$\therefore$ x-intercept: $(8, 0)$

y-intercept, $x=0$

$$\frac{8-x}{x-2} = y \Rightarrow y = \frac{8-0}{0-2}$$

$$y = \frac{8}{-2} = -4$$

y-intercept $(0, -4)$

(iv) On the right branch of the graph, the curve approaches the x-axis ($y=0$).

v) Horizontal asymptote

$$y = \frac{8-x}{x-2}$$

$$f(x) = \frac{ax^n + \dots}{bx^m + \dots}$$

If $n=m$

horizontal asymptote $y = \frac{a}{b}$

$$y = \frac{8-x}{x-2}$$

horizontal asymptote $y = \frac{-1}{1}$

$$y = -1$$

vi) Domain

$$x-2=0$$

$$x=2$$

$$\therefore x < 2 \text{ or } x > 2$$

Interval notation $(-\infty, 2) \cup (2, \infty)$

Range: $y = -1$

$$y < -1 \text{ or } y > -1$$

Interval notation

$$(-\infty, -1) \cup (-1, \infty)$$

10. a) Vertical asymptote
 $x = 0$.

Horizontal asymptote
 $y = 1$.

Horizontal intercepts:

$(-2, 0)$ $(2, 0)$

$$f(x) = \frac{a}{b(x-h)} + k$$

$$a = 1 \quad b = 1$$

$$(h, k) = (0, 1)$$

$$f(x) = \frac{1}{x} + 1 \quad \text{or} \quad \frac{1+x}{x}$$

b) Vertical asymptotes

$$x = 2, \quad x = -7$$

Horizontal asymptote

$$y = -4$$

Horizontal intercepts:

$(-5, 0)$ $(-2, 0)$

$$f(x) = a \frac{(x+5)(x+2)}{(x-2)(x+7)}$$

$$y = -4$$

$$f(x) = \frac{-4(x+5)(x+2)}{(x-2)(x+7)}$$

$$f(x) = \frac{-4(x^2 + 7x + 10)}{x^2 + 5x - 14}$$

$$f(x) = \frac{-4x^2 - 28x - 40}{x^2 + 5x - 14}$$

